Lightest scalar in the $SU_L(2) \times SU_R(2)$ linear σ model

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Abstract

We consider the lightest scalar meson in the frame of the $SU_L(2) \times SU_R(2)$ linear σ model, keeping in mind that this model could be the low energy realization of the two-flavour QCD. We show that the σ field is described by its four-quark component at least in the σ resonance energy (virtuality) region and the $\sigma \to \gamma\gamma$ decay is the four quark transition. We emphasize that residues of the σ pole in the $\pi\pi \to \pi\pi$ and $\gamma\gamma \to \pi\pi$ amplitudes do not give an idea about the σ meson nature, and the progress in studying the σ meson production mechanisms in different processes could essentially further us in understanding its nature.

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The $SU_L(2)\times SU_R(2)$ linear σ model [1]

$$L=\frac{1}{2}\left[(\partial_\mu\sigma)^2+(\partial_\mu\overrightarrow{\pi})^2\right]-\frac{m_\sigma^2}{2}\sigma^2-\frac{m_\pi^2}{2}\overrightarrow{\pi}^2$$
$$-\frac{m_\sigma^2-m_\pi^2}{8f_\pi^2}\left[\left(\sigma^2+\overrightarrow{\pi}\right)^2+4f_\pi\,\sigma\left(\sigma^2+\overrightarrow{\pi}^2\right)\right] \qquad (1)$$

had played an outstanding part in the making of hadron physics. In principle, it could be the low energy realization of the two-flavour QCD. Up to now this model is an excellent laboratory for elucidating subtle points in low energy hadron physics. In particular, we showed [2] that in the linear σ-model there is a negative background phase which hides the σ meson, so that the $\pi\pi$ scattering phase shift does not pass over 90^0 at putative resonance mass. It has been made clear that shielding of wide lightest scalar mesons in chiral dynamics is very natural. This idea was picked up (see, for example, Refs. [3, 4]) and triggered a new wave of theoretical and experimental searches for the σ and κ mesons, see Particle Data Group Review [5]. Below we use our results [2] to analyze the σ meson production in the $\gamma\gamma$ collisions.

Using the simplest Dyson equation for the isoscalar scalar $\pi\pi$ scattering amplitude with the real intermediate $\pi\pi$ states only (in other words, with regard to the real intermediate $\pi\pi$ states only in every rescattering act) we obtained [2] the simple solution, satisfying both unitarity and Adler's self-consistency conditions [6],

$$T_0^0=\frac{T_0^{0(tree)}}{1-i\rho_{\pi\pi}T_0^{0(tree)}}=\frac{e^{2i\delta_0^0}-1}{2i\rho_{\pi\pi}}=T_{bg}+e^{2i\delta_{bg}}T_{res}\,,$$
$$T_0^{0(tree)}=\frac{m_\pi^2-m_\sigma^2}{32\pi f_\pi^2}\left[5-3\frac{m_\sigma^2-m_\pi^2}{m_\sigma^2-s}-2\frac{m_\sigma^2-m_\pi^2}{s-4m_\pi^2}\ln\left(1+\frac{s-4m_\pi^2}{m_\sigma^2}\right)\right]\,,$$
$$\delta_0^0=\delta_{bg}+\delta_{res}\,,\quad \rho_{\pi\pi}=\sqrt{1-4m_\pi^2/s}\,,\quad g_{\sigma\pi^+\pi^-}=-\frac{m_\sigma^2-m_\pi^2}{f_\pi}\,,\quad f_\pi=92.4\,\mathrm{MeV}\,,$$
$$T_{res}=\frac{1}{\rho_{\pi\pi}}\left[\frac{\sqrt{s}\Gamma_{res}(s)}{M_{res}^2-s+\mathrm{Re}\Pi_{res}(M_{res}^2)-\Pi_{res}(s)}\right]=\frac{e^{2i\delta_{res}}-1}{2i\rho_{\pi\pi}}\,,\quad m_\pi=139.6\,\mathrm{MeV}\,,$$
$$T_{bg}=\frac{\lambda(s)}{1-i\rho_{\pi\pi}\lambda(s)}=\frac{e^{2i\delta_{bg}}-1}{2i\rho_{\pi\pi}}\,,\ \lambda(s)=\frac{m_\pi^2-m_\sigma^2}{32\pi f_\pi^2}\left[5-2\frac{m_\sigma^2-m_\pi^2}{s-4m_\pi^2}\ln\left(1+\frac{s-4m_\pi^2}{m_\sigma^2}\right)\right]\,,$$
$$\mathrm{Re}\Pi_{res}(s)=-\frac{g_{res}^2(s)}{16\pi}\lambda(s)\rho_{\pi\pi}^2\,,\quad \mathrm{Im}\Pi_{res}(s)=\sqrt{s}\Gamma_{res}(s)=\frac{g_{res}^2(s)}{16\pi}\rho_{\pi\pi}\,,$$
$$M_{res}^2=m_\sigma^2-\mathrm{Re}\Pi_{res}(M_{res}^2)\,,\quad g_{res}(s)=\frac{g_{\sigma\pi\pi}}{|1-i\rho_{\pi\pi}\lambda(s)|}\,,\quad g_{\sigma\pi\pi}=\sqrt{\frac{3}{2}}\,g_{\sigma\pi^+\pi^-}\,, \qquad (2)$$

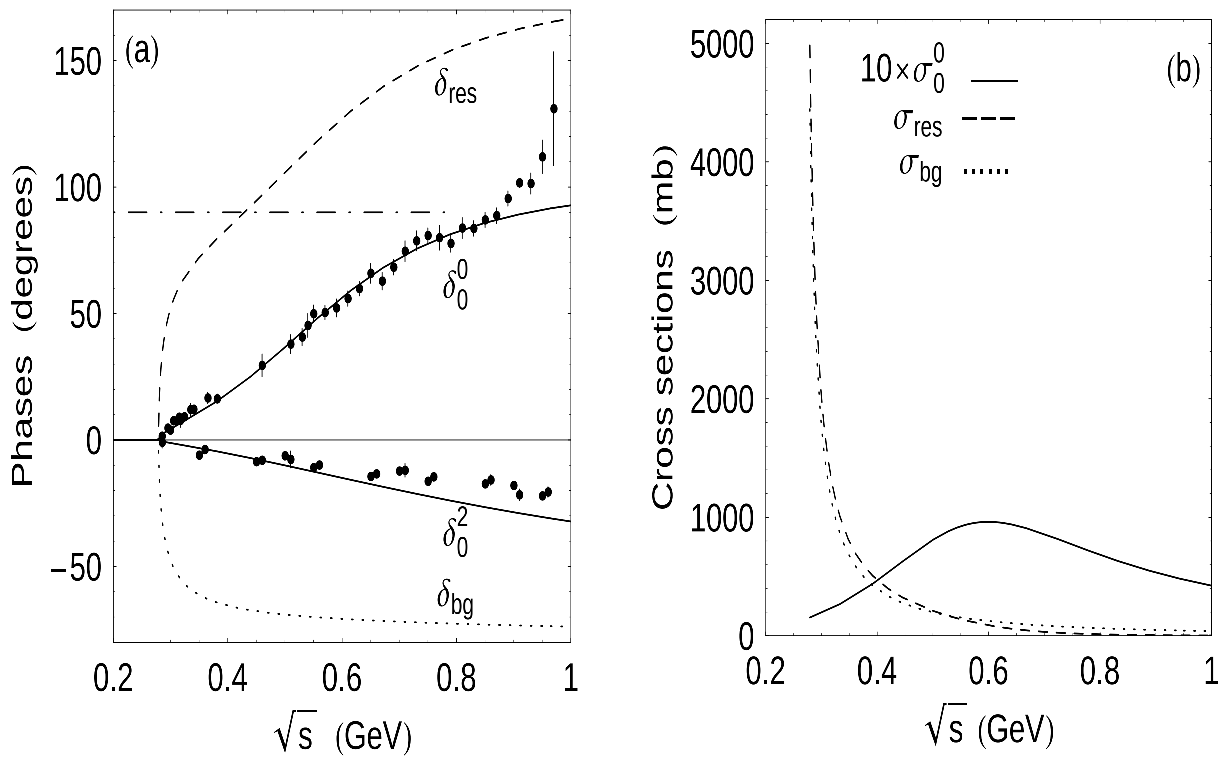

FIG. 1: The σ model, our approximation, the details in the text. The data on δ_0^0 and δ_0^2 from Refs. [23, 24], respectively.

where s is the $\pi\pi$ system invariant mass square. These formulae show that the resonance contribution is strongly modified by the chiral background amplitude.

In a similar manner we find for the isotensor scalar $\pi\pi$ scattering amplitude

$$T_0^2 = \frac{T_0^{2(tree)}}{1 - \imath\rho_{\pi\pi}T_2^{0(tree)}} = \frac{e^{2i\delta_0^2} - 1}{2i\rho_{\pi\pi}},$$
$$T_0^{2(tree)} = \frac{m_\pi^2 - m_\sigma^2}{32\pi f_\pi^2}\left[2 - 2\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2}\ln\left(1 + \frac{s - 4m_\pi^2}{m_\sigma^2}\right)\right]. \tag{3}$$

Our approximation (2) and (3) describes the data acceptably, see Ref. [2] and Fig.1(a), for the bare σ meson mass $m_\sigma = 0.93$ GeV obtained from fitting δ_0^0 in the region $2m_\pi \leq \sqrt{s} \leq 0.87$ GeV. The chiral shielding is demonstrated on Fig.1(a) with the help of the δ_{res}, δ_{bg}, δ_0^0 phases, and on Fig.1(b) with the help of the cross sections

$$\sigma_{res} = \frac{32\pi}{s}\left|T_{res}\right|^2, \qquad \sigma_{bg} = \frac{32\pi}{s}\left|T_{bg}\right|^2, \qquad \sigma_0^0 = \frac{32\pi}{s}\left|T_0^0\right|^2. \tag{4}$$

Our other results are

$$
\begin{aligned}
&M_{res} = 0.43\,\text{GeV}\,, \quad \Gamma_{res}(M_{res}^2) = 0.67\,\text{GeV}\,, \quad m_\sigma = 0.93\,\text{GeV} \\
&\Gamma_{res}^{renorm}(M_{res}^2) = \left(1 + d\text{Re}\Pi_{res}(s)/ds|_{s=M_{res}^2}\right)^{-1}\Gamma_{res}(M_{res}^2) = 0.53\,\text{GeV}\,, \\
&a_0^0 = 0.18\,m_\pi^{-1},\ a_0^2 = -0.04\,m_\pi^{-1},\ (s_A)_0^0 = 0.45\,m_\pi^2\,,\ (s_A)_0^2 = 2.02\,m_\pi^2\,,
\end{aligned} \tag{5}
$$

where a_0^0 and a_0^2 are the scattering lengths, $(s_A)_0^0$ and $(s_A)_0^2$ are the Adler zero positions in T_0^0 and T_0^2, respectively.

The σ pole position

$$
s_R = (0.21 - i0.26) \times \text{GeV}^2, \quad \sqrt{s_R} = M_R - i\Gamma_R/2 = (0.52 - i0.25) \times \text{GeV}\,. \tag{6}
$$

The residues of the σ pole in T_0^0 and T_{res},

$$
\begin{aligned}
&T_0^0 \to \frac{g_\pi^2}{s - s_R}\,, && T_{res} \to \frac{(g_\pi^{res})^2}{s - s_R}\,, \\
&g_\pi^2 = (0.12 + i0.21) \times \text{GeV}^2\,, && (g_\pi^{res})^2 = -(0.25 + i0.11) \times \text{GeV}^2
\end{aligned} \tag{7}
$$

have large imaginary parts. So, considering the residue of the σ pole in T_0^0 as the square of its coupling constant to the $\pi\pi$ channel is not a clear guide to understanding the σ meson nature.

However the amplitudes on the physical axis (5) are rather significant. Let us consider the propagator of the σ field

$$
\frac{1}{D_\sigma(s)} = \frac{1}{M_{res}^2 - s + \text{Re}\Pi_{res}(M_{res}^2) - \Pi_{res}(s)}\,, \tag{8}
$$

which determines T_{res}. The contribution to the σ meson self-energy $\Pi_{res}(s)$ is caused by the intermediate $\pi\pi$ states, that is, by the four-quark intermediate state if we keep in mind that the $SU_L(2) \times SU_R(2)$ linear σ model could be the low energy realization of the two-flavour QCD. This contribution shifts the Breit-Wigner (BW) mass greatly $m_\sigma - M_{res} = 0.50\,\text{GeV}$. So, half the BW mass is determined by the four-quark contribution even if m_σ has another nature, for example, the two-quark one. The imaginary part dominates the propagator modulus in the region 300 MeV $< \sqrt{s} <$ 600 MeV. So, the σ field is described by its four-quark component at least in this energy (virtuality) region [7, 8, 9, 10].

In the field theory approach one has the following S wave $\gamma\gamma \to \pi\pi$ amplitudes, satisfying the unitarity condition, [11], see also [12],

$$\begin{aligned}
&T_S(\gamma\gamma \to \pi^+\pi^-) = T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) + 8\alpha I_{\pi^+\pi^-}\, T_S(\pi^+\pi^- \to \pi^+\pi^-) \\
&= T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) + 8\alpha I_{\pi^+\pi^-}\, \left(\frac{2}{3}\, T_0^0 + \frac{1}{3}\, T_0^2\right) \\
&= \frac{2}{3} e^{i\delta_0^0} \left\{ T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) \cos\delta_0^0 + 8\frac{\alpha}{\rho_{\pi\pi}}\, \mathrm{Re}I_{\pi^+\pi^-} \sin\delta_0^0 \right\} \\
&+ \frac{1}{3} e^{i\delta_0^2} \left\{ T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) \cos\delta_0^2 + 8\frac{\alpha}{\rho_{\pi\pi}}\, \mathrm{Re}I_{\pi^+\pi^-} \sin\delta_0^2 \right\}\,,
\end{aligned} \tag{9}$$

and

$$\begin{aligned}
&T_S(\gamma\gamma \to \pi^0\pi^0) = 8\alpha I_{\pi^+\pi^-}\, T_S(\pi^+\pi^- \to \pi^0\pi^0) = 8\alpha I_{\pi^+\pi^-}\, \left(\frac{2}{3}\, T_0^0 - \frac{2}{3}\, T_0^2\right) \\
&= \frac{2}{3} e^{i\delta_0^0} \left\{ T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) \cos\delta_0^0 + 8\frac{\alpha}{\rho_{\pi\pi}}\, \mathrm{Re}I_{\pi^+\pi^-} \sin\delta_0^0 \right\} \\
&- \frac{2}{3} e^{i\delta_0^2} \left\{ T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) \cos\delta_0^2 + 8\frac{\alpha}{\rho_{\pi\pi}}\, \mathrm{Re}I_{\pi^+\pi^-} \sin\delta_0^2 \right\}\,,
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
&I_{\pi^+\pi^-} = \frac{m_\pi^2}{s} \left(\pi + i \ln \frac{1+\rho_{\pi\pi}}{1-\rho_{\pi\pi}}\right)^2 - 1\,, \quad s \geq 4m_\pi^2\,, \\
&T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) = \frac{8\alpha}{\rho_{\pi^+\pi^-}} \mathrm{Im}I_{\pi^+\pi^-}\,.
\end{aligned} \tag{11}$$

Eqs. (9) and (10) assume that the S wave $\pi\pi$ scattering amplitudes lie on the mass shell in the rescattering loop $\gamma\gamma \to \pi^+\pi^- \to \pi\pi$; $I_{\pi^+\pi^-}$ is the attribute of the triangle diagram $\gamma\gamma \to \pi^+\pi^- \to \sigma$ (or any scalar).

The cross sections are

$$\begin{aligned}
&\sigma_S(\gamma\gamma \to \pi^+\pi^-) = \frac{\rho_{\pi\pi}}{32\pi s} \left|T_S(\gamma\gamma \to \pi^+\pi^-)\right|^2\,, \\
&\sigma_S(\gamma\gamma \to \pi^0\pi^0) = \frac{\rho_{\pi\pi}}{64\pi s} \left|T_S(\gamma\gamma \to \pi^0\pi^0)\right|^2\,.
\end{aligned} \tag{12}$$

As seen from Fig. 2(a), our calculation agrees satisfactorily with the $\gamma\gamma \to \pi^0\pi^0$ data in the σ meson region $\sqrt{s} < 0.8$ GeV. Unfortunately, the $\gamma\gamma \to \pi^+\pi^-$ data are fragmentary in this region, see Fig. 2(b). Nevertheless, our calculation does not contradict them.

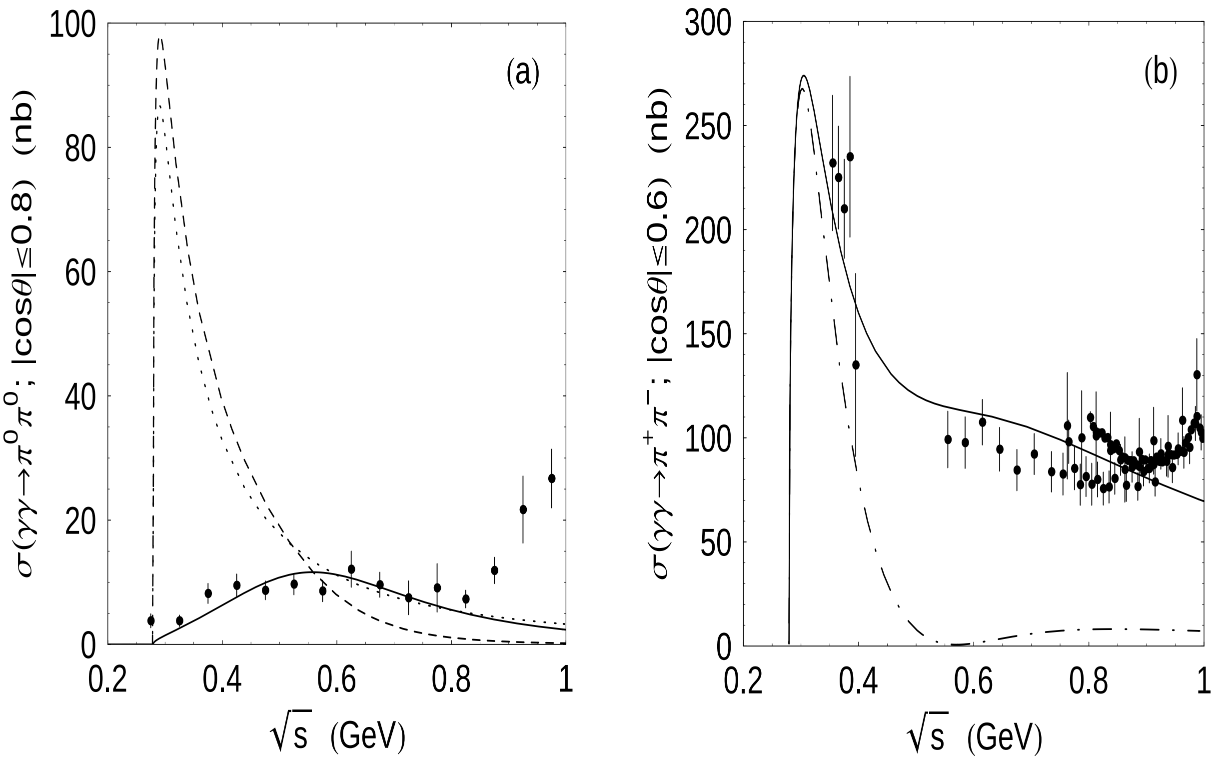

FIG. 2: The σ model, our approximation, the details in the text. (a) The solid, dashed, and dotted lines correspond to $\sigma_S(\gamma\gamma\to\pi^0\pi^0)$, $\sigma_{res}(\gamma\gamma\to\pi^0\pi^0)$, and $\sigma_{bg}(\gamma\gamma\to\pi^0\pi^0)$, respectively. The data from Ref. [25]. (b) The dashed-dotted line corresponds to $\sigma_S(\gamma\gamma\to\pi^+\pi^-)$. The solid line takes into account additionally the contribution of the higher waves in $T^{Born}(\gamma\gamma\to\pi^+\pi^-)$ and their interference with $T_S(\gamma\gamma\to\pi^+\pi^-)$, which takes place for $|\cos\theta|\le 0.6$, see details in Ref. [11]. The data from Refs. [26]

As seen from Eqs. (9) and (10) the shielding of the σ meson takes place in the $\gamma\gamma\to\pi\pi$ amplitudes for the strong destructive interference between T_{res} and T_{bg} as in the $\pi\pi\to\pi\pi$ amplitudes. In Fig.2(a) is it demonstrated with the help of the cross sections

$$\sigma_{res}(\gamma\gamma\to\pi^0\pi^0)=\frac{\rho_{\pi\pi}}{64\pi s}\left|T_{res}(\gamma\gamma\to\pi^0\pi^0)\right|^2=\frac{\rho_{\pi\pi}}{64\pi s}\left|8\alpha I_{\pi^+\pi^-}\times\frac{2}{3}T_{res}\right|^2,$$

$$\sigma_{bg}(\gamma\gamma\to\pi^0\pi^0)=\frac{\rho_{\pi^0\pi^0}}{64\pi s}\left|T_{res}(\gamma\gamma\to\pi^0\pi^0)\right|^2=\frac{\rho_{\pi\pi}}{64\pi s}\left|8\alpha I_{\pi^+\pi^-}\times\frac{2}{3}T_{bg}\right|^2, \tag{13}$$

and $\sigma_S(\gamma\gamma\to\pi^0\pi^0)$.

Let us consider the $\sigma\to\gamma\gamma$ decay

$$g(res\to\pi^+\pi^-\to\gamma\gamma\,,\,s)=\frac{\alpha}{2\pi}I_{\pi^+\pi^-}\times g_{res\,\pi^+\pi^-}(s)\,,$$

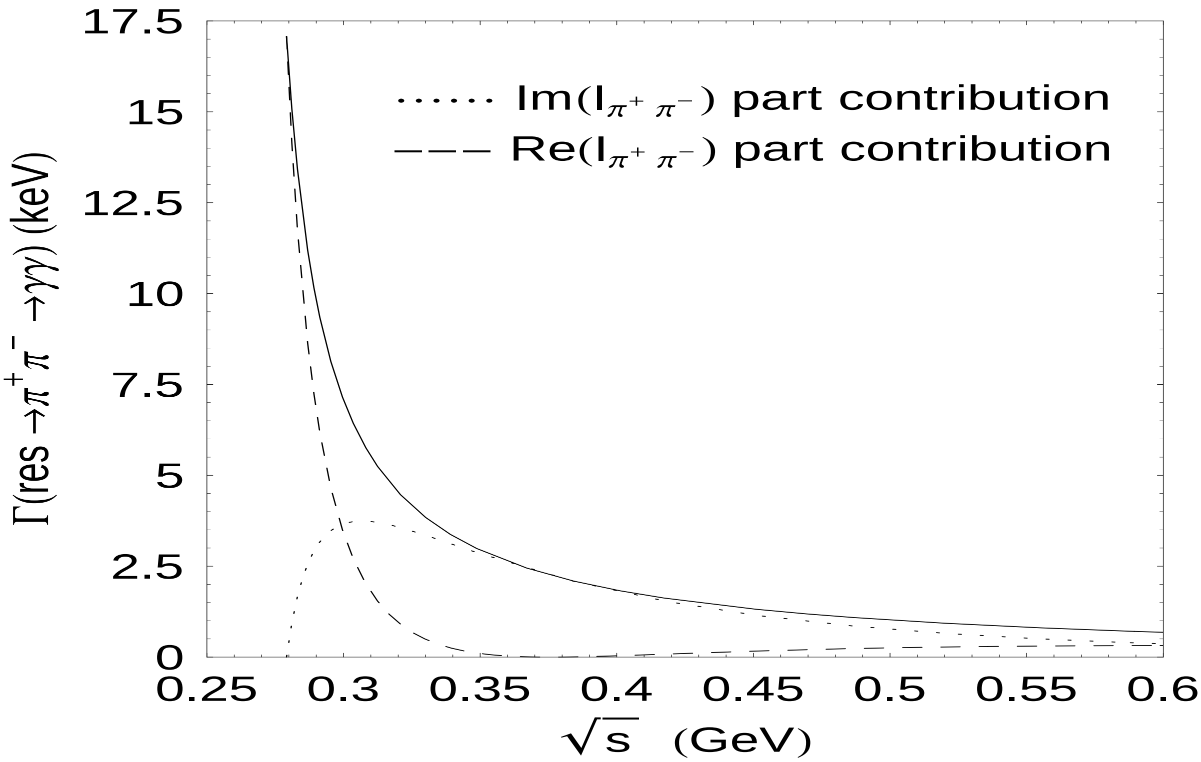

FIG. 3: The σ model. The $\sigma \to \gamma\gamma$ decay in our approximation, the details in the text.

$$\Gamma(res \to \pi^+\pi^- \to \gamma\gamma) = \frac{1}{16\pi\sqrt{s}} \left| g(res \to \pi^+\pi^- \to \gamma\gamma\,,\, s) \right|^2 , \tag{14}$$

where $g_{res\,\pi^+\pi^-}(s) = \sqrt{2/3} \times g_{res}(s) = g_{\sigma\pi^+\pi^-} \Big/ \left|1 - i\rho_{\pi\pi}\lambda(s)\right|$.

So, the the $\sigma \to \gamma\gamma$ decay is described by the triangle $\pi^+\pi^-$-loop diagram $res \to \pi^+\pi^- \to \gamma\gamma$ $(I_{\pi^+\pi^-})$. Consequently, it is the four-quark transition [13] because we imply a low energy realization of the two-flavour QCD by means of the the $SU_L(2) \times SU_R(2)$ linear σ model. As Fig. 3 suggests, the real intermediate $\pi^+\pi^-$ state dominates in $g(res \to \pi^+\pi^- \to \gamma\gamma)$ in the σ region $\sqrt{s} < 0.6$ GeV.

Thus the picture in the physical region is clear and informative. But, what about the pole in the complex s-plane? Does the pole residue reveal [15] the σ indeed?

Taking the $\gamma\gamma \to \pi^0\pi^0$ amplitude normalization like the T_0^0 and T_{res} one we obtain

$$\frac{1}{16\pi}\sqrt{\frac{3}{2}}\, T_S(\gamma\gamma \to \pi^0\pi^0) \to \frac{g_\gamma g_\pi}{s - s_R}\,, \quad \frac{1}{16\pi}\sqrt{\frac{3}{2}}\, T_{res}(\gamma\gamma \to \pi^0\pi^0) \to \frac{g_\gamma^{res} g_\pi^{res}}{s - s_R}\,,$$

$$g_\gamma g_\pi = (-0.45 - i0.19) \times 10^{-3}\,\mathrm{GeV}^2, \quad g_\gamma = (-0.985 + i0.12) \times 10^{-3}\,\mathrm{GeV}^2\,,$$

$$g_\gamma^{res} g_\pi^{res} = (0.53 - i0.13) \times 10^{-3}\,\mathrm{GeV}^2, \quad g_\gamma^{res} = (-0.45 - i0.95) \times 10^{-3}\,\mathrm{GeV}^2,$$

$$g_\gamma/g_\pi = g_\gamma^{res}/g_\pi^{res} = (-1.61 + i1.21)\times 10^{-3}\,,$$

$$\Gamma(\sigma\to\gamma\gamma) = |g_\gamma|^2 \Big/ M_R \approx \Gamma_{res}(\sigma\to\gamma\gamma) = |g_\gamma^{res}|^2 \Big/ M_R \approx 2\,\text{keV}\,. \tag{15}$$

It is interesting to compare ratios $g_\gamma/g_\pi = g_\gamma^{res}/g_\pi^{res}$ with the ratio

$$g\left(res\to\pi^+\pi^-\to\gamma\gamma\,,\,M_{res}^2\right)\Big/\,g_{res}\left(M_{res}^2\right) = (-0.35 + i1.25)\times 10^{-3}\,, \tag{16}$$

which are independent on the different normalization in themselves.

We find it hard to believe that anybody could learn the complex but physically clear dynamics of the $\sigma\to\gamma\gamma$ decay described above from the residues of Eq. (15).

In Ref. [16] was obtained

$$\sqrt{s_R} = M_R - i\Gamma_R/2 = \left(441^{+16}_{-8} - i272^{+12.5}_{-9}\right)\times\text{MeV} \tag{17}$$

with the help of the Roy equation [17].

Our result (6) agrees with the above only qualitatively. This is natural, because our approximation (2) and (3) gives only a semiquantitative description of the data at $\sqrt{s} < 0.4$ GeV. In addition, we do not take into account an effect of the $K\bar{K}$ channel, the $f_0(980)$ meson, and so on; that is, do not consider the $SU_L(3)\times SU_R(3)$ linear σ model.

Could the above scenario incorporates the primary lightest scalar four-quark state [18]? Certainly the direct coupling of this state to $\gamma\gamma$ via neutral vector pairs ($\rho^0\rho^0$ and $\omega\omega$), contained in its wave function, is negligible $\Gamma(q^2\bar{q}^2\to\rho^0\rho^0+\omega\omega\to\gamma\gamma)\approx 10^{-3}$ keV [19]. But its coupling to $\pi\pi$ is strong and leads to $\Gamma(q^2\bar{q}^2\to\pi^+\pi^-\to\gamma\gamma)$ similar to $\Gamma(res\to\pi^+\pi^-\to\gamma\gamma)$ in Fig. 3 [20]. Let us add to Eq. (10) the amplitude for the the direct coupling of σ to $\gamma\gamma$ [22]

$$T_{direct}(\gamma\gamma\to\pi^0\pi^0) = s g^{(0)}_{\sigma\gamma\gamma} g_{res}(s) e^{i\delta_{bg}} \Big/ D_{res}(s)\,, \tag{18}$$

where $g^{(0)}_{\sigma\gamma\gamma}$ is the direct coupling constant of σ to $\gamma\gamma$, the factor s is caused by gauge invariance. Fitting the $\gamma\gamma\to\pi^0\pi^0$ data gives a negligible value of $g^{(0)}_{\sigma\gamma\gamma}$, $\Gamma^{(0)}_{\sigma\gamma\gamma} = \left|M^2_{res} g^{(0)}_{\sigma\gamma\gamma}\right|^2 \Big/ (16\pi M_{res}) \approx 0.0034$ keV, in astonishing agreement with prediction [19].

As already noted in Ref. [8], the majority of current investigations of the mass spectra in scalar channels does not study particle production mechanisms. Because of this, such investigations are essentially preprocessing experiments, and the derivable information is very relative. The progress in understanding the particle production mechanisms could

essentially further us in revealing the light scalar meson nature. We hope that it is shown above clearly.

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[9] Note also that the authors of Ref. [10], using the derivative $\sigma\pi\pi$ coupling in the σ meson propagator, conclude from the QCD sum-rules that a two-quark current does not describe the light and broad sigma meson, quite consistent with the present analysis.

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